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RELIABILITY AND PERFORMANCE ANALYSIS OF CLIENT SERVER NETWORK UNDER k −OUT-OF- n: G WITH COPULA DISTRIBUTION APPROACH

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ABSTRACT
The purpose of this paper is to study reliability measures and analysis of a computer network, that a combination of three subsystems A, B, and C connected in series-parallel configuration, the subsystem A divided into two servers directory server DS and file server FS, the subsystem B stand as switch SW, and subsystem C, our clients, which comprises five subsystems working 3- out-of-5: G policy. The system can fail in two ways: partially, degraded or completely. The systems fail fully due to the failure of any of the following subsystems: A, B, or C. If at least one unit in subsystem A and three units in subsystem C, are operational, the system is said to be partially failed. The model is analyzed using supplementary variables techniques and Laplace transform. General distribution and copula family are used to recover degraded and completely failed states, respectively.

Keywords: Reliability, availability, mean time to system failure, Copula distribution, cost analysis and supplementary variable techniques.

INTRODUCTION

The study of reliability modeling began during World War II in 1939, and since then, significant efforts have been made to develop a thorough theoretical framework for reliability modeling. The discipline is primarily concerned with the requirements and assessments of the probability of a device executing its function adequately for the desired duration under the encountered operating conditions. In recent years, reliability has taken on new dimensions due to the complexity of larger systems and the possibility of failure. In today's technological age, unreliability causes system inefficiency, excessive maintenance, and can even endanger human life. In today's technological world, practically everyone relies on the ongoing operation of a wide range of machinery and equipment for our safety, security, mobility, and financial welfare. We receive our electronic agreements from illuminations, hospital monitoring control, next-generation aircraft, nuclear power plants data exchange system, and aerospace applications to perform whenever we need them. When they fail, the outcomes can be catastrophic, injurious, or even loss of life. The theory of dependability is the scientific discipline that analyses the general regularity that must be maintained during the design, research, manufacture, acceptance, and use of units/components to obtain the highest efficacy of their use.

The most prevalent form of redundancy is known as k-out-of-n redundancy, which is further classified into two categories: k-out-of-n: G and k-out-of-n: F. In k-out-of-n: G redundancy, the system requires at least k out of n units to be operational for successful functioning. If fewer than k units are operational, the system fails. On the other hand, k-out-of-n: F redundancy means that if k units out of n have failed, the entire system is considered to have failed. Numerous authors have extensively studied system reliability. Singh *et al.* (2021) examined the performance assessment of complex repairable systems with n identical units under the k-out-of-n: G scheme and a copula linguistic repair approach. Abubakar and Singh (2019) investigated the assessment and performance of industrial systems using the Gumbel-Hougaard copula approach. Singh *et al.* (2009) explored reliability characteristics for Internet data centers with redundant servers, including a primary mail server. Additionally, Lirong and Haijun (2014) Analytical method for reliability and MTTF assessment of coherent systems with dependent components discussed the reliability of systems with various failure modes and common cause failures under a preemptive resume. Park (2015) investigated a multi-component system with imperfect repair during warranty periods using renewal processes. Zhang (2019) conducted an analysis on computer network reliability, focusing on intelligent cloud computing methods. Nagiya *et al.* (2017). examined a tree topology network environment analysis under reliability approach the analysis of tree topology network environments from a reliability perspective, incorporating nonlinear factors. Muhammad *et al.* (2020) conducted a cost-benefit analysis of three different configurations of series-parallel dynamo systems. Dillon *et al.* (1994) explored the analysis of common causes of failure in k-out-of-n: G systems, which consist of repairable units. Yusuf (2015) evaluated the performance of repairable systems considering minor deterioration under imperfect repair conditions.

Yen *et al.* (2016) conducted a study on the reliability and sensitivity analysis of a controllable repair system with warm standbys and working breakdowns. Singh *et al.* (2018) assessed the performance and cost of a repairable complex system consisting of two subsystems connected in a series configuration. Singh and JyotiGulati (2015) investigated the performance assessment of a computer center at Yobe State University in Nigeria under various policies using copula methods. Ibrahim *at el.* (2021) investigated the availability and cost implications of a complex tree topology in computer networks featuring multiple servers, employing the Gumbel-Hougaard family method. Geon Yoon, Hyun *et al.* (2006). delved into the implementation of ring topology-based redundancy Ethernet for industrial networks. Kumar *et al.* (2020) conducted a Probabilistic assessment of complex system with two subsystems in series arrangement with multi-types failure and two types of repair using copula. D. R. Cox, (1995). The analysis of non-markov stochastic processes by the inclusion of supplementary variables Yusuf and Hussaini (2014) analyzed a three-unit redundant system with three types of failures and general repair mechanisms. Negi and Singh (2015) examined the reliability characteristics of a non-repairable complex system connected in series.

Singh *et al.* (2018) conducted a study focusing on the performance assessment of repairable systems arranged in a series configuration, examining various failure and repair policies utilizing copula linguistics. While prior research has explored different models to enhance the understanding of complex systems' performances and availability, the majority of studies have concentrated on treating complex repairable systems as undergoing a single repair between two contiguous transition states. In this particular investigation, the authors analyzed multiple reliability measures of a complex repairable system consisting of three subsystems, employing a k-out-of-n: G configuration with two types of repair. The system comprised three subsystems: A, B, and C. Subsystem A housed a directory server (DS) and a file server (FS), while B encompassed the switch, and C served multiple clients. Additionally, the system included five units operating under a 3-out-of-5: G policy. Both series and parallel arrangements of the system were scrutinized. The study utilized the Gumbel Hougaard family copula distribution for calculation and illustration purposes.
In this system, $\{S_0\}$ represents a fully operational state, while {S1, S2, S3, S4, S5} denote states of partial failure or degradation, and {S6, S7, and S8} indicate complete failure states. General repair is employed to restore degraded states, while the

Gumbel-Hougaard family copula is used for complete failures. Supplementary variables and Laplace transformations are utilized for analyzing system reliability measures, including availability, reliability, mean time to failure (MTTF), and cost analysis, which are presented using tables and graphs. The reliability measures of this complex repairable system, which involves three subsystems and adopts a k-out-of-n: G configuration with two types of repair, have been thoroughly examined. The system is organized into three subsystems: A, B, and C. Subsystem A comprises a directory server (DS) and a file server (FS), while subsystem B accommodates the switch. Subsystem C serves multiple clients and incorporates five units within its setup.

MATERIALS AND METHOD

State	State Description
$\{S_0\}$	The state S0 represents a condition where all subsystems are in perfect working order, indicating an optimal operational state.
$\{S_1\}$	The state S1 signifies a degraded condition wherein subsystem C experiences partial failure, attributed to the malfunction of one unit within subsystem C.
$\{S_2\}$	In state S2, another degraded condition is observed, characterized by a major failure within subsystem C, resulting from the malfunction of two units within subsystem C. The system is undergoing general repair to address these issues.
$\{S_3\}$	In state S3, there is a degraded condition with partial failure occurring in subsystem A, attributed to the failure of either the directory server (DS) or the file server (FS) within subsystem A.
$\{S_4\}$	In state S4, there is a partial failure state resulting from the malfunction of one unit from subsystem A (either DS or FS) and one unit from subsystem C. The system is currently undergoing general repair to address these issues.
$\{S_5\}$	In state S5, there is a degraded condition with a major partial failure occurring in subsystem C, caused by the malfunction of one unit from subsystem A (either DS or FS) and two units from subsystem C.
$\{S_6\}$	In state S6, the system is in a complete failed state, resulting from the failure of three units within subsystem C. The system is undergoing repair utilizing copula.
$\{S_7\}$	In state S7, the system is in a complete failed state due to the failure of subsystem A, specifically the failure of both the directory server (DS) and the file server (FS). The system is currently undergoing repair utilizing copula.
$\{S_8\}$	In state S8, the system is in a complete failed state as a result of the failure of subsystem B, specifically the failure of the switch (SW). The system is undergoing repair utilizing copula.

The state description outlines that S0 signifies a perfect state wherein both subsystems are functioning properly. States S1, S2, S3, S4, and S5 represent operational states where the system is functioning, albeit with varying degrees of performance degradation. On the other hand, states S6, S7, and S8 denote completely failed states where the system is non-operational.

ASSUMPTIONS

Themodel's assumptions are discussed below.

1) At the outset, all subsystems are assumed to be in optimal working order.

2) For the system to be operational, it necessitates the activation of at least one unit from subsystem A, along with three units from subsystem C, in

conjunction with the activation of subsystem B.
3) If subsystem B fails and at least three units fail in subsystem C, the system will enter a complete state of failure.
4) If all subsystems A, B, and C fail, it will result in damage to the entire system.
5) A failed unit of the system can be repaired when it is in an operative or failed state.
6) Units of the system that have failed can be repaired regardless of whether they are in an operative or failed state.
7) Repairs are conducted following both a general distribution and a copula distribution.
8) The assumption is made that a repaired system functions identically to a new system, and no damage occurs during the repair process.
9) Once the failed unit is repaired, it becomes immediately ready to perform its designated task.

Notations

t	The time variable is represented on a time scale.
S	A variable for Laplace transform for all expressions is typically denoted as s.
λ_f/λ_d	The failure rates of servers within subsystem A
λ_{sw}	The failure rates of servers within subsystem B
λ_c	The failure rates of servers within subsystem C
λ_f, λ_{sw} and λ_d	The failure rates of servers within subsystem A, B, and C respectively
$\phi(x)$	The repair rates for degraded states of all subsystems.
$\mu_0(x), \mu_0(y)$	Repair rates for complete failed states.
$P_i(x,t)$	The probability that the system is in state Si at instant 's', where i ranges from 0 to 8, can be represented using a probability vector. Each element of the vector corresponds to the probability of the system being in the respective state at the given instant.
$P_i(s)$	Laplace transformation of state transition probability P (t).
$E_p(t)$	Expected profit over the time interval [0, t).
K_1, K_2	Revenue and service cost per unit time over the interval [0, t) respectively.
$S_\phi(x)$	Standard repair distribution function $S_\phi(x) = \phi(x)e^{\int_0^\infty \phi(x)d(x)}$
$u_0(x) = C_\theta(u_1(x), u_2(x))$	The expression of joint probability failed state Si to according to Gumbel Hougaard family copula is given as $C_\theta\big(u_1(x), u_2(x)\big) = exp[x^\theta + log\phi(x)]^{\frac{1}{\theta}}$ where, $u_1 = \phi(x)$ and $u_2 = \exp$ where θ as a parameter, 1≤ θ≤ ∞.

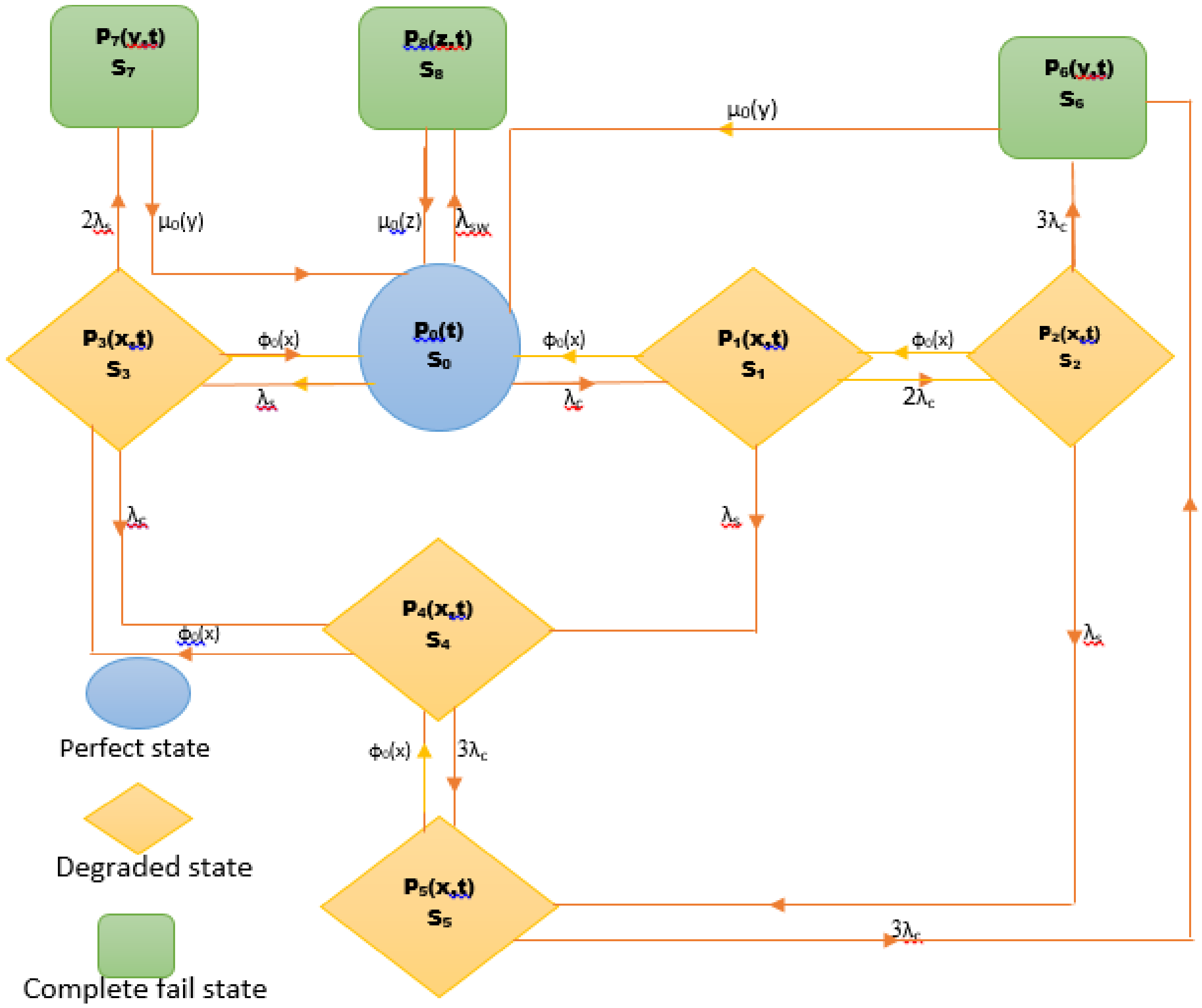

Figure 1: State Transition Diagram

FORMULATION OF MATHEMATICAL MODEL

The authors derived the following set of differential equations based on the literature review and by employing the methodology utilized by V.V. Singh et al. in [1].

$$\left(\frac{\partial}{\partial t}+\lambda_f+\lambda_c+\lambda_{sw}\right)P_0(t)=\int_0^\infty \Phi(x)P_1(x,t)dx+\int_0^\infty \Phi(x)P_3(x,t)dx+$$
$$\int_0^\infty \mu_0(y)P_6(y,t)dx+\int_0^\infty \mu_0(y)P_7(y,t)dy+\int_0^\infty \mu_0(z)P_8(z,t)dz, (1)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_f+2\lambda_c+\Phi(x)\right)P_1(x,t)=0, (2)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_f+3\lambda_c+\Phi(x)\right)P_2(x,t)=0, \qquad (3)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_f+\lambda_d+\lambda_c+\Phi(x)\right)P_3(x,t)=0, (4)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_c+\Phi(x)\right)P_4(x,t)=0, (5)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}++3\lambda_c+\Phi(x)\right)P_5(x,t)=0, (6)$$

$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+ +3\lambda_c \ +\Phi\ (x)\right)P_5(x,t)=0,$ (7)
$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\mu_0(y)\right)P_6(y,t)=0,$ (8)
$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial \alpha}+\mu_0(y)\right)P_7(y,t)=0,$ (9)
$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(z)\right)P_8(z,t)=0.$ (10)

Boundary conditions are:
$P_1(0,t)=\lambda_c P_0(t)$,(11)
$P_2(0,t)=2\lambda_c\lambda_c P_0(t)$,(12)
$P_3(0,t)=\lambda_f P_0(t)$,(13)
$P_4(0,t)=2\lambda_f\lambda_c P_0(t)$,(14)
$P_5(0,t)=6(\lambda_f\lambda_c\lambda_c)P_0(t)$,(15)
$P_6(0,t)=6\lambda_c(\lambda_c\lambda_c+\lambda_f\lambda_c\lambda_c)P_0(t)$,(16)
$P_7(0,t)=2\lambda_f\lambda_d P_0(t)$,(17)
$P_8(0,t)=\lambda_{sw}P_0(t)$.(18)

SOLUTION OF THE MODEL

Taking Laplace transformation of equations (1)-(18) with the help of initial conditions$P_0(0)=1$ and $P_0(1)=0$, we obtained the following equations below:
$(s+\lambda_f+\lambda_c+\lambda_{sw})\overline{P_0}(s)=\int_0^\infty \Phi(x)\overline{P_1}(s)(x,s)dx+\int_0^\infty \Phi(x)\overline{P_3}(x,s)dx+\int_0^\infty \mu_0(y)\overline{P_6}(y,s)dy+\int_0^\infty \mu_0(y)\overline{P_7}(y,s)dy+\int_0^\infty \mu_0\overline{(z)P_8}(z,s)dz$,(19)
$\left(s+\frac{\partial}{\partial x}+\lambda_f+2\lambda_c+\Phi_0(x)\right)\overline{P_1}(x,s)=0$,(20)
$\left(s+\frac{\partial}{\partial x}+\lambda_f+3\lambda_c+\Phi_0(x)\right)\overline{P_2}(x,s)=0$,(21)
$(s+\frac{\partial}{\partial x}+\lambda_f+\lambda_d+\lambda_c+\Phi_0(x))\overline{P_3}(x,s)=0$,(22)
$\left(s+\frac{\partial}{\partial x}+2\lambda_c \ +\Phi\ (x)\right)\overline{P_4}(x,s)=0$,(23)
$\left(s+\frac{\partial}{\partial x}+3\lambda_c \ +\Phi_0(x)\right)\overline{P_5}(x,s)=0$,(24)
$\left(s+\frac{\partial}{\partial x}+\mu_0(y)\right)\overline{P_6}(y,s)=0$,(25)
$\left(s+\frac{\partial}{\partial x}+\mu_0(y)\right)\overline{P_7}(y,s)=0$,(26)
$\left(s+\frac{\partial}{\partial x}+\mu_0(z)\right)\overline{P_8}(z,s)=0$,(27)

The Laplace transformations of the boundary conditions are:
$\overline{P_1}(0,s)=\lambda_c\overline{P_0}(s)$,(28)
$\overline{P_2}(0,s)=2\lambda_c\lambda_c\overline{P_0}(s)$,(29)
$\overline{P_3}(0,s)=\lambda_f\overline{P_0}(s)$,(30)
$\overline{P_4}(0,s)=2\lambda_f\lambda_c\overline{P_0}(s)$, (31)
$\overline{P_5}(0,s)=6(\lambda_f\lambda_c\lambda_c)\overline{P_0}(s)$,(32)
$\overline{P_5}(0,s)=6(\lambda_f\lambda_c\lambda_c)\overline{P_0}(s)$,(33)
$\overline{P_6}(0,s)=6\lambda_c(\lambda_c\lambda_c+\lambda_s\lambda_c\lambda_c)\overline{P_0}(s)$,(34)
$\overline{P_7}(0,s)=2\lambda_f\lambda_d\overline{P_0}(s)$,(35)

$\overline{P_8}(0,s) = \lambda_{sw}\overline{P_0}(s).$ (36)

Now solving equations (20)-(36) with the help of equations (11)-(19), yields,

$\overline{P_0}(s) = \frac{1}{D(s)}$,(37)

$\overline{P_1}(s) = \frac{\lambda_c}{D(s)}\left\{\frac{1-S_\phi(S+\lambda_f+2\lambda_c)}{S+f+2\lambda_c}\right\},$ (38)

$\overline{P_2}(s) = \frac{2\lambda_c}{D(s)}\left\{\frac{1-S_\phi(S+\lambda_f+3\lambda_c)}{S+\lambda_f+3\lambda_c}\right\},$ (39)

$\overline{P_3}(s) = \frac{\lambda_f}{D(s)}\left\{\frac{1-S_\phi(S+\lambda_f+\lambda_d\lambda_c)}{S+\lambda_f+\lambda_d+\lambda_c}\right\},$ (40)

$\overline{P_4}(s) = \frac{2\lambda_f\lambda_c}{D(s)}\left\{\frac{1-S_\phi(S+2\lambda_c)}{S+2\lambda_c}\right\},$ (41)

$\overline{P_5}(s) = \frac{6(\lambda_f\lambda_c\lambda_c)}{D(s)}\left\{\frac{1-S_\phi(S+3\lambda_c)}{S+3\lambda_c}\right\},$ (42)

$\overline{P_6}(s) = \frac{6\lambda_c[\lambda_c\lambda_c+\lambda_s\lambda_c\lambda_c]}{D(s)}\left\{\frac{1-S\mu_0(x)}{S}\right\},$ (43)

$\overline{P_7}(s) = \frac{3\lambda_f}{D(s)}\left\{\frac{1-S\mu_0(x)}{S}\right\},$ (44)

$\overline{P_8}(s) = \frac{\lambda_{sw}}{D(s)}\left\{\frac{1-S\mu_0(x)}{S}\right\},$ (45)

$\overline{P_{up}}(s) = \overline{P_0}(s) + \overline{P_1}(s) + \overline{P_2}(s) + \overline{P_3}(s) + \overline{P_4}(s) + \overline{P_6}(s),$ (46)

$\overline{P_{down}}(s) = 1 - \overline{P_{up}}(s)$,(47)

Where,

$$D(s) = \left\{(S+\lambda_f+\lambda_c+\lambda_{sw}) - \left(\begin{array}{c}\frac{\lambda_c\phi}{s+\lambda_f+2\lambda_c+\phi} + \frac{\lambda_f\phi}{s+\lambda_c+\lambda_f+\lambda_d+\phi} \\ + \frac{6\lambda_c((\lambda_c)^2+(\lambda_f\lambda_c)^2)\mu_0(x)}{s+\mu_0(x)} + + \frac{3\lambda_f\mu_0(x)}{s+\mu_0(x)} + \frac{\lambda_{sw}}{s+\mu_0(x)}\end{array}\right)\right\}. \quad (48)$$

The $\bar{P}_{up}(s)$and $\bar{P}_{down}(s)$ are the system Laplace transform of the state probabilities in operative and Failed state and have the relation $\bar{P}_{up}(s) + \bar{P}_{down}(s) = 1$. Hence, we have the following results: $\bar{P}_{up}(s) = \sum_{i=0}^{5} p_i(s)$ and $\bar{P}_{down}(s) = 1 - \bar{P}_{up}(s)$.

$$\bar{P}_{up}(s) = \left\{\begin{array}{c}1 + \frac{\lambda_c}{S+\lambda_f+2\lambda_c+\phi} + \frac{2\lambda_c{}^2}{S+\lambda_f+2\lambda_c+\phi} \\ + \frac{\lambda_f}{S+\lambda_c+\lambda_f+\lambda_d+\phi} + 2\left(\frac{2\lambda_f\lambda_c}{S+\lambda_f+2\lambda_c+\phi}\right)\end{array}\right\}. (49)$$

Numerical Results of the Proposed Model

<u>Availability Analysis</u>

By Setting $S_{\mu_0}(s) = \bar{S}_{exp\left[x^\theta+(\log\phi(x))^\theta\right]^{\frac{1}{\theta}}}(s) = \frac{exp\left[x^\theta+(\log\phi(x))^\theta\right]^{\frac{1}{\theta}}}{s+exp\left[x^\theta+(\log\phi(x))^\theta\right]^{\frac{1}{\theta}}}$ and $S_{\phi_s}(s) = \frac{\phi_s}{s+\phi_s}$.

The expression of availability is obtained by taking the inverse Laplace transform of equation (49) together with the values of failure rates, $\lambda_s = 0.0001$, $\lambda_c = 0.0002$,$\lambda_d = 0.0003$,$\lambda_{sw} = 0.0004$and $\phi(x) = \theta = x = 1$ and $\mu_0(x) = \mu_0(y) = 2.781$.

$$D(x) = \left\{\begin{array}{l}0.0001601810230e^{-2.783444983t} - 0.001176172766e^{-1.001081612t} \\ +0.00007207456685e^{-1.000620895t} + 1.000943917e^{-0.007152509170t}\end{array}\right\} (50)$$

The values of $\bar{P}_{up}(t)$ through variation of time t= 0, 10, 20, 30, 40, 50, 60, 70, 80, 90 shown in Table 1 and figure 2.*Table 1: Variation of Availability with respect to time (t)*

Time(t)	Availability
0	1
10	0.9318
20	0.8675
30	0.8076
40	0.7518
50	0.6999
60	0.6516
70	0.6066
80	0.5648
90	0.4895

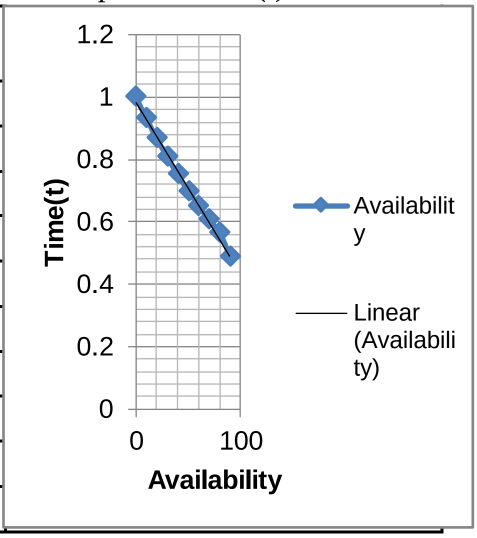

Figure 2 *Variation of Availability with respect to time (t)*

Reliability Analysis

Setting all repair rates to zero with the same value of failure and repair rates in equation (49), ϕ(x)andμ₀(x)and $\lambda_s = 0.0001$, $\lambda_c = 0.0002$, $\lambda_d = 0.0003$, $\lambda_{sw} = 0.0004$, and then taking inverse Laplace transform, we obtained the expression of reliability.

$$R(t) = \left\{ \begin{array}{l} 0.040000000000e^{-0.000500000000000t} + 0.9189041096e^{-0.00800000000000t} \\ +0.04109589041e^{-0.000700000000000t} + 1.333398106e^{-0.001100000000t} \end{array} \right\} \quad (51)$$

Marked at different values of time *t*= 0, 10, 20, 30, 40, 50, 60, 70, 80, 90 units of time, the value of Reliability is shown in Table 2. and Figure. 3.

Table 2: *Variation of Reliability with respect to time (t)*

Time(t)	Reliability
0	1
10	0.9288
20	0.8631
30	0.8024
40	0.7464
50	0.6946
60	0.6468
70	0.6026
80	0.5618
90	0.5241

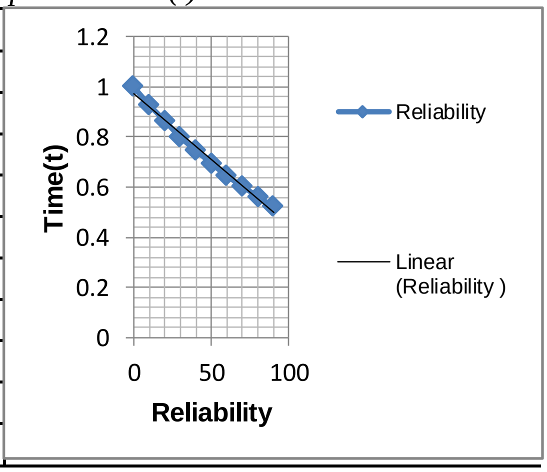

Figure 3: *Variation of Reliability with respect to time (t)*

Mean Time To Failure (MTTF) Analysis

The expression for MTTF is found by taking all repairs to be zero in equation (49), and set the limit of s tends to zero:

$$MTTF = \lim_{s\to 0}\overline{P}_{up}(s) = \frac{1}{\lambda_s+\lambda_c+\lambda_d}\left\{1+\frac{\lambda_s}{\lambda_c+2\lambda_d}+\frac{2\lambda_c^2}{\lambda_s+2\lambda_d}+\frac{\lambda_d}{\lambda_s+\lambda_d}+\frac{4\lambda_c\lambda_d}{\lambda_s}\right\}. \quad (53)$$

Setting $\lambda_s = 0.0001$, $\lambda_c = 0.0002$, $\lambda_d = 0.0003$, $\lambda_{sw} = 0.0004$, and changing λ_s, λ_c, λ_d and λ_{sw}, respectively as, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, in (53), the variation of mean time to failure is found with respect to failure rates as shown in Table 3 and corresponding Figure 4.

Table 3: *MTTF with failure rate*

Failure Rate	MTTF Λs	MTTF λc	MTTF λw
0.01	145.48	10.72	12.13
0.02	74.82	10.7	11.34
0.03	50.65	10.68	10.7
0.04	38.44	10.67	10.16
0.05	31.08	10.65	9.71
0.06	26.156	10.64	9.32
0.07	22.63	10.63	8.99
0.08	19.98	10.62	8.7
0.09	17.91	10.61	8.44

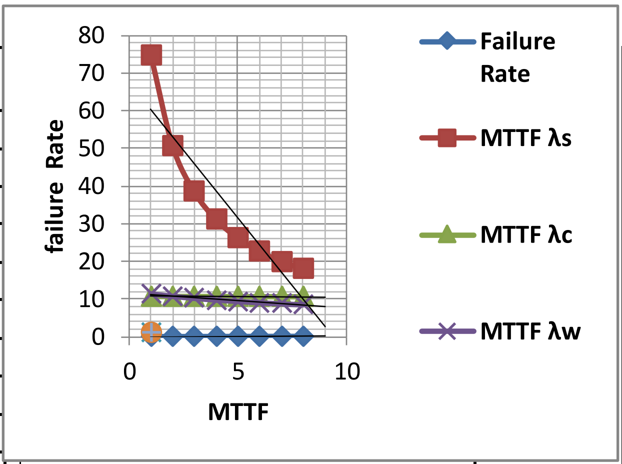

Figure 4: Variation of MTTF with Failure rates

Cost Analysis

The predictable profit over the time interval [0, t), can be estimate by the following relation

$$E_p(t) = K_1\int_0^t P_{up}(t)dt - K_2 t\,. \quad (54)$$

If the service facility of the system is always available, where k_1 is revenue generated and k_2 service cost per unit time. For the same set of the parameter of failure and repair rates in (49), the expression of cost benefit analysis is obtained. By fixing the revenue $k_1 = 1$ and taking the $k_2 = 0.5, 0.4, 0.3, 0.2, 0.1$ respectively together with the variation of $t = 0, 10, 20, \ldots, 90$, units of time, the results for expected profit was obtained as shown in Table 4 and Figure 5.

Table 4: Cost benefit

Time (t)	$E_P(t)$: K_2=0.5	$E_P(t)$: K_2=0.4	$E_P(t)$: K_2=0.3	$E_P(t)$: K_2=0.2	$E_P(t)$: K_2=0.1
0	0	0	0	0	0
10	4.66	5.66	6.66	7.66	8.66
20	8.65	10.7	12.7	14.7	16.7
30	12	15	18	21	24
40	14.8	18.8	22.8	26.8	30.8
50	17.1	22.1	27.1	32.1	37
60	18.8	24.8	30.8	36.8	42.8
70	20.1	27.1	34.1	41.1	48.1
80	21	29	37	45	53
90	21.5	30.4	39.4	48.4	57.4

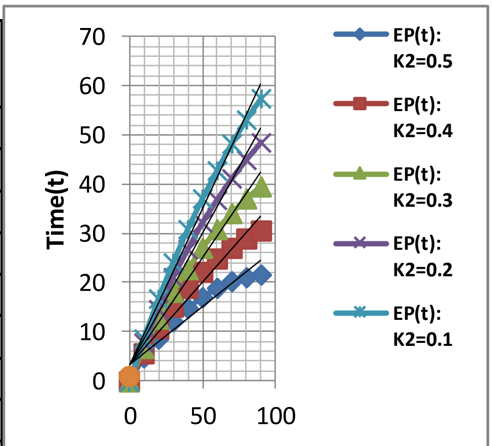

Figure5: Expected profit figure

DISCUSSION AND CONCLUSION

This paper examined system performance analysis of client server network under k −out-of- n: G with copula distribution approach in terms of various types of failure values. The system performance was assessed through reliability measures for various values of failure and repair rates. Table 1 and Figure 2 demonstrate the availability of the complex over time, with fixed failure rates at λs = 0.0001, λc = 0.0002, λd = 0.0003, and λsw = 0.0004. The system's availability gradually decreases as the probability of failure increases, ultimately approaching zero as time progresses. However, one can reliably predict the future behavior of the complex system at any stage given a set of parametric values. Table 2 and Figure 3 illustrate the system's reliability in the absence of repair. The figures clearly indicate that the reliability of the system declines more rapidly compared to availability. This observation underscores the importance of repairs in enhancing the system's performance.

Table 3 and Figure 4 evaluate the mean time to failure of the system (MTTF) concerning the variation of failure rates. The changes in MTTF directly correlate with the system's reliability. MTTF computations were conducted for different values of failure rates: λs, λc, λd, and λsw. From the figure, it is evident that the variation in MTTF corresponding to failure rates λs is higher compared to other failures, indicating that the system is less affected by fluctuations in these values. Table 4 and figure 5 reveal the information on how the profit has been generated, by fixing revenue cost per unit time K_1= 1, and varies the service costs K_2 = 0.5, 0.4, 0.3, 0.2 and 0.1, if we examine critically from Figure 5 we can discloses that the expected profit increases for low service cost. Which finally shows the Networking system is reliable.

CONCLUSION

Discussion and Concluding Remark

This paper examined system performance in terms of various types of failure values $\beta_1 = 0.01$, $\beta_2 = 0.02$, $\beta_{H1} = 0.03$ and $\beta_{H2} = 0.04$, Two types of repair are used in the associated elements of distributed systems: copula repair and general repair. The availability variation over time was represented in Table 2 and Figure 4. Figure 2 depicts how availability decreases as failure rates increase when a copula repair policy is used. Table 3 and Figure 5 show the availability of the system when the repair follows the typical distribution pattern. Table 4 and Figure 6 show the evolution of reliability over time. When a system's failure rate increases, the system's reliability decreases over time. Tables 4 and 6 show that reliability has lower values than availability in Tables 2 and 3.The mean-time-to-failure (MTTF) of the system is shown in Table 5 and Figure 7 as a function of failure rate variation β_1, β_2, β_{S1} and β_{S2} when all other parameters are held constant. The variation in MTTF corresponding to β_1, β_2, β_{S1} and β_{S2} are very close. According to this analysis, the failure rate β_1, β_2, β_{S1} and β_{S2} are more responsible for the system's successful operation. Table 6 and Figure 8 show the results of the sensitivity analysis examined in this study.

REFERENCES

Abubakar M.I. & Singh, V.V. (2019). Performance assessment of industrial system African textile manufacturers, Ltd. through copula approaches, *Journal of Operations Research and Decisions*, doi.org/: 10.37190/ord19040.

Cox, D. R. (1995). The analysis of non-markov stochastic processes by the inclusion of supplementary variables, Proc. Comb. Phil. Soc. Math. Phys. Sci. 51, 433-444.

Dhillon, B.S. & Anude, O.C. (1994). Common cause failure analysis of a k-out-of-n: G system with pairable unit, *Microelectronics Reliability*.34(3), 429-442.

Ibrahim, K.H., Isa, M.S., Yusuf, I. & Tukur, I. (2021). Availability and cost analysis of complex tree topology of computer network with Multiserver using Gumbel-Hougaard family copula approach, R T AND A. 16(1), 101-116.

Isa, M.S., Ali, U.A., Yusuf, B & Yusuf, I. (2020). Cost benefit analysis of tree different series parallel dynamo configurations, Life Cycle *Reliability and Safety Engineering*. 9(1), 413-423.

Kumar, P., Ibrahim, K.H., Abubakar, M.I. & Singh, V.V. (2020). Probabilistic assessment of complex system with two subsystems in series arrangement with multi-types failure and two types of repair using copula, *Strategic System Assurance and Business Analytics.*

Lado, A., Singh, V.V.,Ibrahim, K.H. & Yusuf, I. (2018). Performance and cost assessment of repairable complex system with two subsystems connected in series configuration, *International Journal of Reliability and Applications*.19(1), 22-42.

Lirong C. & Haijun L. (2014). Analytical method for reliability and MTTF assessment of coherent systems with dependent components, *Reliability Engineering & System Safety*. 92(3), 300-307.

Minjae P., (2015). Warranty cost analysis for multi-component systems with imperfect repair, *International Journal of Reliability and Applications*. 5(1), 51-64.

Nagiya, K., Ram, M. & Dua, A.K. (2017). A tree topology network environment analysis under reliability approach, Nonlinear Studies. *Reliability and Safety Engineering* 24(1), 199-202.

Negi, S. & Singh, S.B, (2015). Reliability analysis of non-repairable complex system with weighted subsystems connected in series, *Applied Mathematics and Computations*. 262(1), 79-89.

Singh, V.V., Ayagi, H.I. & Goel, C.K. (2018). Performance assessment of repairable system in series configuration under different types of failure and repair policies using copula linguistics *International Journal of Reliability and Safety*. 12(4).

Singh, V.V., Abubakar M.I. & Kabiru H.I. (2021). Performance assessment of complex repairable system with n identical unit under *k*-out-of-*n*: G scheme and copula linguistic repair approach, *International Journal of Quality Reliability Management.*

Singh, V.V., Singh, S. B. & Goel, C. K. (2009). Stochastic analysis of internet data center, International J*ournal of Mathematical Sciences Engineering Application*. 3(4), 231-24.

Singh, V.V. & Gulati, J. (2015). Performance assessment of computer centre at Yobe State University Nigeria under Different repair policies using copula, International conference on energy, environment and chemical engineering. (ICEECE2015) ISBN: 978-1-60595-257-4.

Tseng- C., Yen, Wu- Lin C. & Jia-Yu Chen, (2016). Reliability and sensitivity analysis of the controllable repair system with warm standby and working breakdown, Computers and Industrial Engineering.97(3), 84-97.

Yoon, G., Hyun, D., Soon, K., Kwon, C., Oon, Y. and Joon, Y. (2006). Ring Topology-based Redundancy Ethernet for Industrial Network, Siceicase *International Joint Conference*. 1404-1407.

Yusuf, I. & Hussaini, N, (2014). A comparative analysis of three-unit redundant systems with three types of failures', *Arabian Journal for Science and Engineering*. 39(4), 3337-3349.

Zhang, F, (2019). Research on reliability analysis of computer network based on intelligent cloud computing method, *International Journal of Computers and Applications*. 41(4), 283-288.