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EVALUATING THE IMPLICATIONS OF HEAT SOURCE AND SINK ON MHD-FREE CONVECTION FILLED WITH POROUS MATERIAL IN AN UPSTANDING CHANNEL

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ABSTRACT
This paper performs an analytical simulation on the implications of magnetic field and porosity effect on an electrically conducting fluid in an upward-looking channel influenced by heat source and sink and the Navier slip condition. The governing equations have been derived by using the theory of simultaneous Ordinary Differential Equations. The effects of the pertinent flow parameters on velocity and temperature were graphically displayed. The frictional force and the heat transfer coefficient on the heated and cold plates have also been calculated. It is revealed that the hydrodynamic and thermodynamic gradients of the fluid are enhanced by the heat generation parameter ($H_t < 0$) and permeability of the porous materials (Da), whereas raising the heat absorption parameter ($H_t > 0$) and magnetic number (M) makes the fluid velocity diminish. The comparison of the present investigation with previously published work demonstrates an excellent relationship for the limiting cases, thereby verifying the accuracy and validity of the present work.

Keywords: Heat generation/absorption, Magneto-hydrodynamics (MHD), Darcy number (Da), Navier Slip, Vertical channel

INTRODUCTION

Natural convection flow in a boundary layer regime is a motion that is produced from the interaction of gravity with density differences within a fluid. These differences happen because of the temperature or concentration gradients or due to their composition. Studies pertaining to free convection flows in vertical parallel plate channels for a single phase of an incompressible viscous fluids have gained increasing attention over the years, both theoretically (exact or approximate solutions) and experimentally, owing to the fact that many practical applications involve natural convection heat transfer (Al-Subaie and Chamkha, 2004; Zullkifree *et al*. 2019). The laminar flow of a viscous fluid between two parallel surfaces, one of which travels tangentially with respect to the other, is known as the Couette flow. This flow can be motivated by a pressure gradient that is present in the flow direction (Mng'ang'a 2023). In view of these, Khan *et al.* (2022) used the second law to evaluate how Newtonian heating affects the Couette flow of a viscoelastic dusty fluid and the transfer of heat in a rotating frame. Umavathi *et al.* (2010) studied the Poiseuille-Couette flow with heat transfer in the inclined channel. They came to the conclusion that increases in the Gasthof number, angle of inclination, and height ratio increase velocity, while increases in the Hartmann number, viscosity, and conductivity ratios lower the velocity profiles. Beg *et al.* (2010) analyzed the oblique magnetic field in an oscillating, extremely porous medium. Barikbin *et al.* (2014) examined non-Newtonian fluid for MHD Couette flow using the Ritz-Galerkin method. Sreekala and Reddy (2014) studied the effect of an inclined magnetic field on the steady MHD Couette flow. Joseph *et al.* (2014) evaluated heat transfer in an inclined magnetic field with transient hydromagnetic Couette flow between two infinitely parallel porous plates. The temperature-dependent transient MHD Couette flow and heat transfer of dusty fluid were reported by Mosayebidorcheh *et al.* (2015). Jha *et al.* (2015) inspected thermal radiation with unsteady MHD free convective Couette flow. Ngiangia and Okechukwu (2016) pondered the impact of variable electro-conductivity and radiation. They indicated that increases in electroconductivity, Prandtl number, Reynolds number, and Grashof number result in an increase in velocity distribution, while increases in the magnetic field result in a drop in velocity. Raju *et al.* (2016) discussed the influence of diffusion thermodynamics and thermal diffusion on a natural convection Couette flow using the six- element method. Ali *et al.* (2016) investigated convective cooling of the nano fluids in a rotating system. Job and Gunakala (2016) investigated the thermal radiation for upstanding permeable plates using Galerkin's finite element method. Also, Ali *et al.* (2017) considered the Couette flow of a Maxwell fluid in three dimensions with periodic injection or suction. Hussain *et al.* (2018) analyzed instability of the MHD Couette flow of an electrically conducting fluid. Anyanwu *et al.* (2020) discussed the influence of radioactive and a uniform pressure gradient on transient MHD Couette flow. Ajibade *et al.* (2021) outlined the implications of viscous dissipation on steady natural convection Couette flow of a heat-emitting fluid in an up-facing channel.

Basically, the concept of natural convection flows in parallel plate is most often modeled under the assumptions of constant surface temperature, ramped wall temperature, or constant surface heat flux (Rajput 2011a,2011b, Narahari 2012, Singh and Sarehu 2015). However, in many real-life circumstances where the heat transfer from the surface is assumed to be proportional to the local surface temperature, the above assumptions fail. These kind of flows are referred to as conjugate convective flows, and the corresponding condition of the heat transfer to the local surface temperature is called as Newtonian heating (Lesnic *et al.* 2004). Natural convection involving Newtonian heating or cooling through various geometrical shapes and channels have been reported by several scholars. The natural convection boundary layer flow over a vertical surface with Newtonian heating was first studied by Merkin (1994). In view of this, Shehzad *et al.* (2014) investigated three-dimensional flow of Jeffrey fluid with Newtonian heating. Akbar *et al.* (2013) discussed the effect of Newtonian heating on the mixed convective magneto-hydrodynamics peristaltic flow of Jeffrey fluid, while Hamza (2016) investigated the influence of Navier Slip and Newtonian heating in the transient flow of an exothermic fluid in vertical channel. Lesnic *et al.* (1999, 2000) studied free convection boundary layer flows along vertical and horizontal surfaces in a porous medium generated by Newtonian heating. Subsequently, a free convection boundary layer flow above a nearly horizontal surface in a porous medium with Newtonian heating was reported by Lesnic *et al.* (2004). Abid *et al.* (2013) studied heat and mass transfer past an oscillating vertical plate with Newtonian heating, where the governing equations of the problem were solved by the Laplace transform technique. Subsequently, Abid *et al.* (2014) studied an unsteady boundary layer MHD-free convection flow in a porous medium with constant mass diffusion and Newtonian heating analytically using Laplace transform technique. An unsteady hydro-magnetic natural convection flow past an impulsively moving vertical plate with Newtonian heating in a rotating system was studied by Seth *et al.* (2015). Das *et al.* (2015) performed a numerical analysis on unsteady heat and mass transfer of hydro-magnetic Casson fluid across a vertical plate in the presence of chemical reaction and thermal radiation. Qayyum *et al.* (2017) described the heat and mass transfer of water-B nano fluid across a stretching sheet in the coexistence of chemical reactions and thermal radiation. Hayat *et al.* (2018) discussed the impact of MHD flow of Powell-Eyring fluid by a stretching cylinder with thermal radiation through an inclined magnetic field under the Newtonian heating effect. Zin *et al.* (2018) presented an exact and numerical solutions for the unsteady MHD heat and mass transfer flow of Jeffrey fluid flowing through an oscillatory vertical plate provoked by thermal radiation and Newtonian heating factors.

Understanding the impact of heat sink or source on fluid flow is very key in several engineering, technological sectors of exothermic or endothermic fluid response, and these effects are essential in gauging heat transference. Heat source/sink impact have drawn much interest attributable to its possible advantages in fields such as drying engineering processes, chemical factories, coolers and heaters of electrical and mechanical equipment, storage of nuclear waste etc. (Gambo *et al.* 2021). Several scholars looked into the impacts of heat source and sink fluid on several flow regions. With these concerns in mind, Ojemeri *et al*. (2024) recently published an analytical investigation of a chemically reactive hydromagnetic fluid affected by heat generation and absorption flowing across a super hydrophobic micro channel that is heated alternatively. Ojemeri and Onwubuya (2023a) describe the analysis of steady mixed convection flow of Arrhenius-controlled chemical reaction and an exothermic fluid along an isothermally heated super hydrophobic micro channel due to the heat source or sink. Ojemeri and Hamza (2022) presented a theoretical investigation of a chemically reactive fluid in a MHD natural convection flow blended with heat source and sink effects employing a homotopy perturbation approach. Oni and Jha (2019) investigated the impacts of heat source and sink fluid on free convection confined to an upstanding annulus with time-periodic heating boundary conditions. It is concluded that heat source and absorption parameters affect velocity distribution, periodic temperature profile, Nusselt number, and frictional force at the cylinder walls, respectively. Chamkha *et al.* (2017) scrutinized the impact of heat sink or source on hydromagetic mixed convection flow in a porous enclosure filled with a Cu-water nano fluid affected by partial slip conditions. Chu *et al*. (2020) studied the nonlinear thermal radiation and heat sink/source of a bidirectional, periodically non-stationary surface instigated by a rate-type nano fluid involving a gyrotactic microbe. Zhao *et al*. (2021) outlined the impacts of heat source/sink, thermal radiation, and joule heating on the mixed convective entropy-optimized nano material hydromagnnetic flow of Ree-Eyring fluid restricted by two rotating disks.

The consequences of heat generation/absorption, MHD, and Darcy permeability along upward-looking channel using the regular perturbation method have not been investigated in a single work in any of the above-mentioned literature, which prompted the interest in this current study. Thus, motivated by the above knowledge gap, the purpose of this research is to expand the work carried out by Zulkiflee *et al*. (2019) by investigating the impacts of heat generation/absorption, MHD and permeability parameters on a Couette flow inside two vertical parallel plates with the involvement of Newtonian heating using the theory of simultaneous ordinary differential equation. Various illustrative graphs have been sketched to demonstrate the flow pattern of the pertinent parameters embedded in the flow regime. This research can find relevance in engineering and industrial technologies, namely cooling of atomic reactors, geothermal supplies, porous solids drying, thermal insulation, gas drainage, plasma physics, gas turbines, fossil fuel combustion, food processing industries, and so forth.

MATERIALS AND METHOD

We consider a fully developed laminar MHD flow of an electrically conducting viscous fluid passing through two vertical parallel plates with Newtonian heating instigated by heat generation/absorption and Darcy porosity effects. As shown in Figure 1, the wall at $y_0 = 0$ is instigated by Navier slip effect, whereas the no-slip surface is kept at $y_0 = H$. The flow is affected by a uniform transverse magnetic field in the involvement of thermal buoyancy effects. All the fluid characteristics are assumed to be constant. The flow variables are functions of space y only. Following Zullkifree *et al*. (2019), the leading equations for the current problems in dimensional, employing the Boussinesq buoyancy approximation with boundary conditions and assuming that the fluid is affected by heat generation, absorption and porosity effects, can be modeled as follows:

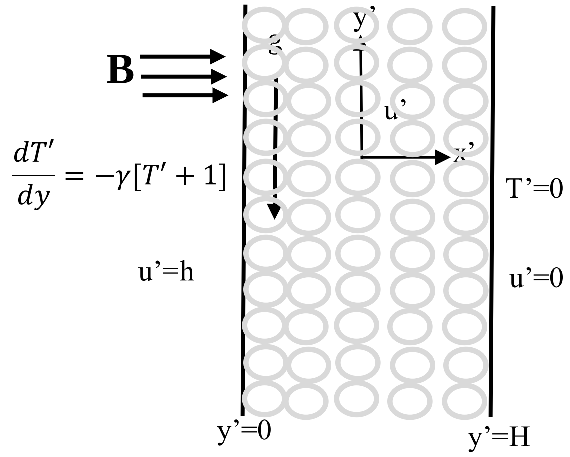

Figure 1: Schematic diagram of the Flow

$$v\frac{d^2u'}{dy'^2} + g\beta(T' - T_O) - \left(\frac{\sigma B_0^2}{\rho} + \frac{1}{K}\right)u' = 0 \quad (1)$$

$$\frac{k}{\rho c_\rho}\frac{d^2T'}{dy'^2} - \frac{Q_0}{\rho c_\rho}(T' - T_O) = 0 \quad (2)$$

Where y' and x' are the dimensional distances along and perpendicular to the plate. u' and T' are the dimensional velocity and temperature. Q_0, ν, k, ρ, c_p, β and g are the dimensional heat generating/absorbing parameter, kinematic viscosity, thermal conductivity, density, specific heat at constant pressure, thermal expansion coefficient, and acceleration due to gravity of the fluid respectively.

The appropriate boundary conditions of this formulation are:

$$\left.\begin{array}{l} \left.\begin{array}{r} u' = U_0 \\ \frac{dT'}{dy'} = -\frac{h_s}{k}T' \end{array}\right\} at\ y' = 0 \\ \left.\begin{array}{r} u' = 0 \\ T' = T_h^0 \end{array}\right\} \qquad at\ y' = h \end{array}\right\} \quad (3)$$

To solve eqns (1) to (3), we employ the dimensionless quantities and parameters:

$$u = \frac{u'}{U}, y = \frac{y'}{h}, \theta = \frac{T' - T_0}{T_w - T_o}, x = \frac{x'v}{Uh^2},$$
$$M^2 = \frac{\sigma\beta_o^2h^2}{\rho v}, Gr = \frac{g\beta(T_w - T_0)h^3}{v^2},$$
$$Da = \frac{k}{h^2}, H_t = \frac{Q_0h^2}{k} \quad (4)$$

inserting the dimensionless quantities in Eqn (4), on the basic eqns (1) to (3), we have the dimensionless governing equation as:

$$\frac{d^2U}{dy^2} - \left(M^2 + \frac{1}{Da}\right)U = -Gr\theta \quad (5)$$

$$\frac{d^2\theta}{dy^2} - H_t\theta = 0 \quad (6)$$

The corresponding boundary conditions are:

$$\left.\begin{array}{l} \left.\begin{array}{r} \frac{d\theta}{dY} = -\gamma(\theta + 1) \\ U = 1 \end{array}\right\} at\ Y = 0 \\ \left.\begin{array}{r} \theta = 0 \\ U = 0 \end{array}\right\} at\ Y = 1 \end{array}\right\} \quad (7)$$

Where M is the magnetic field intensity, Gr is thermal Grashof number, Da is the Darcy porosity parameter, H_t is the Heat source/sink parameter and γ is the Navier slip parameter.

Method of Solution

The resultant ordinary differential equations are systems of ordinary differential equations with constant coefficients. This system of linear ordinary differential equations is solved in closed form by the theory of simultaneous ordinary differential equations (Jha *et al*., 2014). Also, the derived solutions in terms of temperature, velocity, skin friction, and Nusselt number are discussed in detail with the aid of plotted graphs using a user-friendly software program (MATLAB 2015a).

The analytical solutions of the temperature and velocity have been determined as follows:

$$\theta = W_1\cosh\left(y\sqrt{H_t}\right) + W_2\sinh\left(y\sqrt{H_t}\right) \quad (8)$$

$$U = W_3\cosh(Ry) + W_4\sinh(Ry) + W_5cosh\left(y\sqrt{H_t}\right) + W_6sinh\left(y\sqrt{H_t}\right) \quad (9)$$

The rate of heat transfers in terms Nusselt number and the skin friction have been derived as:

$$\frac{d\theta}{dy}\Big|_{y=0} = W_2\sqrt{H_t} \quad (10)$$

$$\frac{d\theta}{dy}\Big|_{y=1} = W_1\sqrt{H_t}\sinh(\sqrt{H_t}) + W_2 \quad (11)$$

$$\frac{dU}{dy}\Big|_{y=0} = W_4\,\mathrm{R} + W_6\sqrt{H_t} \quad (12)$$

$$\frac{dU}{dy}\Big|_{y=1} = W_3\,R\sinh(Ry) + W_4\,R\cosh(Ry) + W_5\sqrt{H_t}sinh(\sqrt{H_t}) + W_6\sqrt{H_t}cosh(\sqrt{H_t}) \quad (13)$$

Where

$$R^2 = M^2 + \frac{1}{Da}, \quad W_1 = \frac{-\gamma/\sqrt{H_t}sinh(\sqrt{H_t})}{\frac{\gamma}{\sqrt{H_t}sinh(\sqrt{H_t})} - cosh(\sqrt{H_t})},$$

$$W_2 = \frac{-\gamma(W_1+1)}{\sqrt{H_t}}, W_3 = 1 - W_5$$

$$W_4 = \frac{W_5[\cosh(R) - cosh(\sqrt{H_t})] - W_6 sinh(\sqrt{H_t}) - \cosh(R)}{\sinh(R)}, W_5 = \frac{-GrW_1}{H_t - R^2}, W_6 = \frac{-GrW_2}{H_t - R^2}.$$

RESULT AND DISCUSSION

The theoretical investigation of heat generation/absorption effect on steady hydro-magnetic flow of an incompressible electrically-conducting fluid traveling vertically upward through two parallel vertical plates saturated with porous material. The formulated governing equations are systems of ordinary differential equations with constant coefficients. This system of linear ordinary differential equations was solved in closed form by the theory of simultaneous ordinary differential equations. Also, the derived solutions in terms of temperature, velocity, skin friction, and Nusselt number have been discussed in detail with the aid of plotted graphs. The default values chosen for this research are$\gamma = 1, M = 0.5, H_t = 0.5, Da = 0.5\ and\ Gr = 100$except otherwise stated, as it relate to real-life situations.

Figures 2a and 2b exhibit the action of heat source or sink parameter on the temperature profile. It is evident from these figures that the temperature is expanded for $H_t < 0$, as portrayed in Figure 2a, but the temperature is reduced for $H_t > 0$, as displayed in Figure 2b. With heat generation along the lower plate, higher temperature profiles are anticipated. Naturally, it is true because when heat is absorbed, the fluid becomes heavier and the convection current decays, leading to a drop in the fluid temperature. On the other hand, raising the heat generation parameter, strengthens the convection current, consequently leading to a weakening in fluid density and a surge in temperature.

Figures 3a and 3b show the effects of changing the heat sink or source settings on the velocity gradient. As expected, the same effect observed in the velocity gradient is replicated in the temperature gradient when the heat source $(H_t < 0)$ and heat sink $(H_t > 0)$ levels increase, as depicted respectively in Figures 3a and 3b. Naturally speaking, this is so because of the additional heat boost, which amplifies the heat flow behavior of the system, thereby giving rise to the thermal profile of the fluid and ultimately enhancing the flow in the system. Additionally, heat source/sink parameter is assumed to carry more heat near the plate, which helps the fluid move faster and expands the fluid's velocity and temperature within the boundary layer region. Also, the thickness of the thermal and momentum boundary plates is stronger in the micro-channel as the internal heat source/sink parameter increases.

Figure 4 explains the action of the Darcy porosity parameter on the fluid velocity. It is clear from this figure that the velocity fluid is propelled as the porosity parameter is increased. Further, by uplifting the permeability of the porous medium, the frictional force decays; as a result, the velocity gradient of the fluid is encouraged.

Figure 5 showcases the function of MHD on the velocity gradient. The pattern demonstrates a decrease in the fluid flow as the magnetic field intensity increases. This is so attributable to the Lorentz force, which appears when a magnetic field produces an electrically conducting fluid and a drag force is created. As a result, the fluid movement dwindle near the plate so that the Lorentz force consequently peters out when the fluid comes to rest.

Figures 6a and 6b showcase the functions of heat generation and absorption on the heat transfer coefficient versus the Navier slip effect. It is demonstrated from figure 6a that at the wall $(y =$

0), the rate of heat transfer decays for mounting level of heat absorption parameter while a counter attribute happens at the wall ($y = 1$) as displayed in figure 6b. However, on the hand, it is noteworthy to report that growing values of heat generation parameter speed up the fluid motion at $y = 1$ as shown in figure 6b, whereas a reverse case is recorded at the lower plate.

Figures 7a and 7b demonstrate the variation of heat sink and source parameter on the skin friction against the Navier slip effect. As shown in figure 7a, increasing the heat emission parameter weakens the drag force at $y = 0$, while increasing it at $y = 1$. The implications of mounting the heat generation parameter is seen to lowers the shear stress at $y = 0$ but a reverse case occurs at$y = 1$. The permeability effect on the frictional force at both plates is sketched against Newtonian heating parameter in Figures 8a and 8b. It can be seen that uplifting the values of the permeability parameter strengthens the frictional force at the plate ($y = 0$), whereas an opposite trend happens at $y = 1$. Figure 9a and 9b display the function of magnetic field on the shear stress. It was discovered that MHD has a diminishing influence on the skin friction at $y = 0$, while a counter behavior is witnessed at $y = 1$.

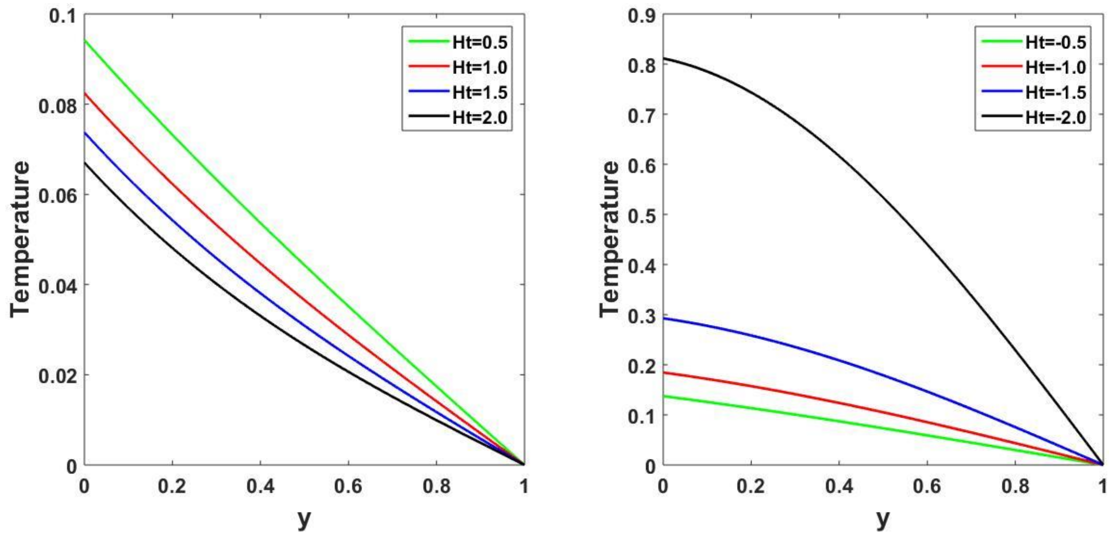

Figure 2. Action of (a) Heat absorption and (b) Heat generation on the Temperature distribution

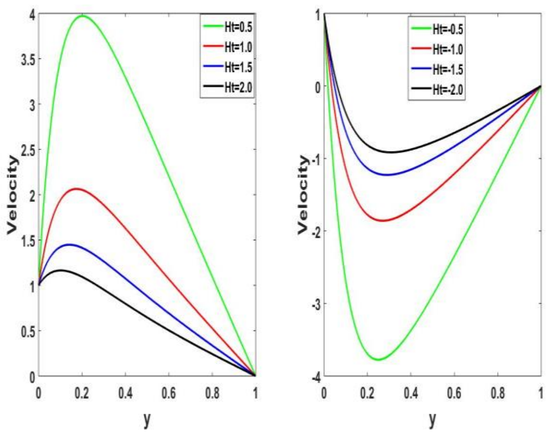

Figure 3. Action of (a) Heat absorption and (b) Heat generation on the Velocity distribution

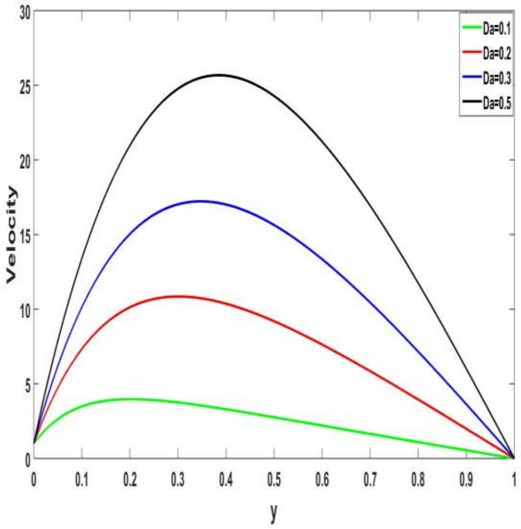

Figure 4. Action of Darcy number on the Velocity distribution

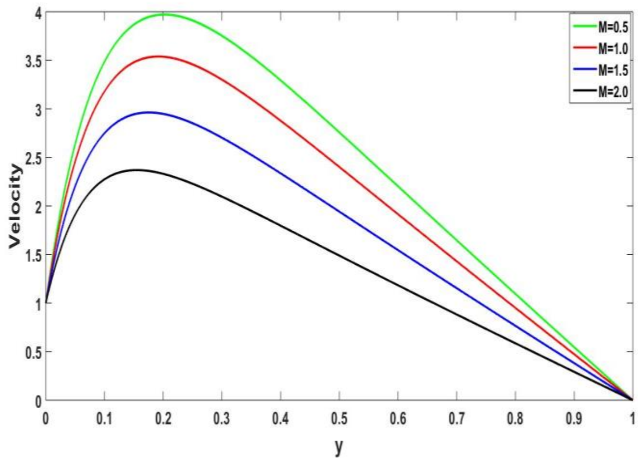

Figure 5. Action of Magnetic number on the Velocity distribution

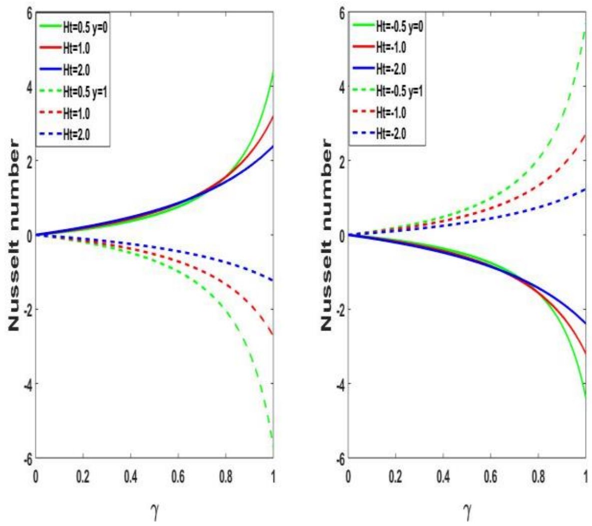

Figure 6. Action of (a) Heat absorption and (b) Heat generation on the Nusselt number at both plates

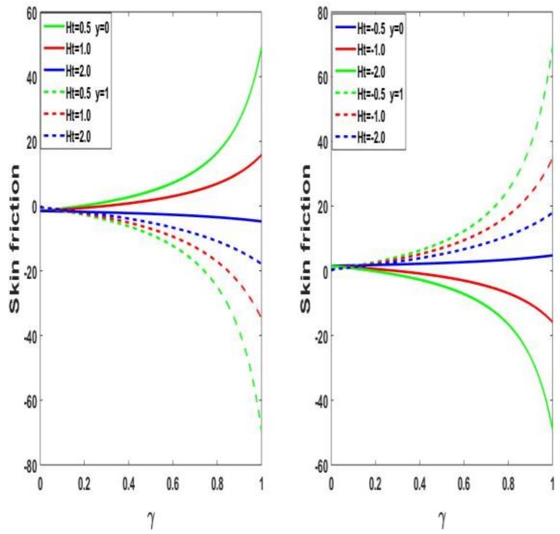

Figure 7. Action of (a) Heat absorption and (b) Heat generation on the Skin friction at both plates

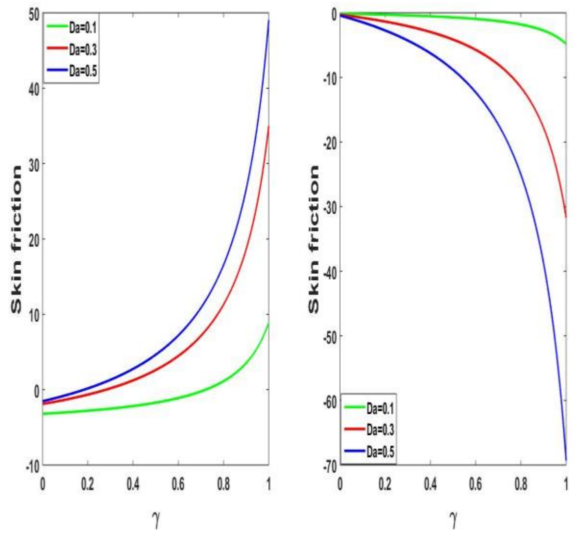

Figure 8. Action of Darcy number on the Skin friction at (a) at $y = 0$ and (b) at $y = 1$

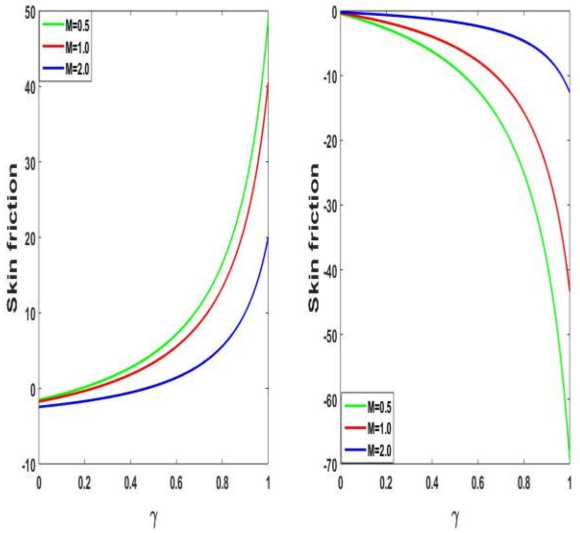

Figure 9. Action of Magnetic number on the Skin friction at (a) at $y = 0$ and (b) at $y = 1$

Validation of Results

The work of Zulkifree *et al*. (2019) is retrieved successfully by setting H_t and M to be zero and Da to be 1000, so that the term $\frac{1}{Da}$ disappears. The validation confirms an excellent agreement between the present research and their work. Table 1 presents the numerical computations of the comparison between the present analysis and their work.

Table 1: Numerical comparison of the current study on the temperature and velocity profiles with the work of Zulkiflee *et al*. (2019), for $\gamma = 0.01$, setting Da to be 1000 and when $H_t = M = 0$.

Y	Zulkiflee *et al*. (2019) θ (Y)	U(Y)	Present work θ(Y)	U(Y)
0.1	0.0091	0.9003	0.0091	0.9003
0.2	0.0081	0.8006	0.0081	0.8006
0.3	0.0071	0.7007	0.0071	0.7007
0.4	0.0061	0.6006	0.0061	0.6006
0.5	0.0051	0.5006	0.0051	0.5006

CONCLUSION

The effect of heat sink or source on the steady hydromagnetic flow of an electrically conducting fluid traveling upward along two parallel plate filled with porous medium has been performed under the influence of thermal Grashof parameter. A semi-analytical method was employed to determine the solutions for temperature, velocity, rate of heat transfers, and sheer stress. The impacts of controlling parameters embedded in the flow configuration are explained in detail with the help of various graphs. Internal heat generation, absorption and permeability effects are vital in most lubrication industries. The findings from this study can be applicable to engineering and technological industries for geothermal systems, porous solids drying, gas drainage, plasma physics, gas turbines, fossil fuel combustion, food processing industries and so on. The summary of the main findings from this research is highlighted below:

i. Taking $H_t < 0$, which denotes heat generation, the fluid velocity and the temperature distributions respectively are observed to significantly improve, whereas for $H_t > 0$ representing heat absorption, a reverse behavior is noticed.

ii. The velocity gradient peters out upon raising the levels of the magnetic parameter due to the Lorentz forces, which manifest under the MHD effect.

iii. Upon increasing the Darcy permeability parameter, drag forces diminish at $y = 0$, whereas a growing effect is noticeable at the upper plate.

iv. Increasing the heat source parameter profoundly improves the drag force and heat transfer rate, respectively at the plate $y = 1$, while the opposite behavior is recorded at $y = 0$. However, converse effects are all observed for rising levels of heat sink parameter for the both plates.

v. By raising the values of magnetic number, the skin friction is opposed at the heated plate, whereas at the cold plate, an opposite trend is recorded.

vi. In the future, this research will be extended to study the effect of exothermic chemical reactions.

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